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Anomaly cancellation and the gauge group of the standard model in NCG

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It is well known that anomaly cancellation *almost* determines the hypercharges in the standard model. A related (and somewhat more stronger) phenomenon takes place in Connes' NCG framework: unimodularity (a technical condition on elements of the algebra) is *strictly* equivalent to anomaly cancellation (in the absence of right-handed neutrinos); and this in turn reduces the symmetry group of the theory to the standard $SU(3) \times SU(2) \times U(1)$.

1. Introduction

There is a deep relationship between anomaly cancellation and the actual values of the hypercharges in the standard model; it is well known [8] that anomaly cancellation only allows for two solutions: the "physical" one, and a "bizarre" solution, with all the hypercharges zero except for the $\bar{u}$ and $\bar{d}$, whose sum must vanish.

One of the many fascinating aspects of the Connes(-Lott) approach to the standard model through Non-Commutative Geometry (NCG) is that the demand of anomaly cancellation is fulfilled through a mathematical restriction on the elements of the algebra, technically called *unimodularity* (somewhat similar to the restriction to unit determinant elements in a unitary group).

It is remarkable that a quite subtle *quantum* property, such as anomaly cancellation, is achieved automatically after imposing an apparently unrelated and much simpler condition, the unimodularity condition. Our purpose in this paper is to explore this relationship. After a short reminder of the NCG set-up, we shall explore the converse property, namely, to what extent is true that anomaly cancellation does imply unimodularity. We shall find that, under certain conditions, *they are strictly equivalent* so that one can say, slightly

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overstating it, that anomaly cancellation, in the NCG context, determines the gauge group in the representation observed in Nature.

2. Connes' unimodularity conditions

The "old scheme" for the NCG reconstruction of the SM has been spelled out in [1, 2, 3]. Recently Connes introduced a "new scheme" [4]. The key element of both is an associative algebra (the noncommutative spacetime) represented by operators on the Hilbert space $\mathcal{H} \oplus \bar{\mathcal{H}}$ of all fermions. The noncommutative gauge potential and gauge field are forms on the noncommutative space, defined via successive commutation with a Dirac-Yukawa operator. We need to consider only one quark and lepton family at a time. Thus, when no right handed neutrinos are present:

$$\begin{aligned} \mathcal{H} &= \mathcal{H}_\ell \oplus \mathcal{H}_q \\ &:= L^2(S) \otimes \begin{pmatrix} \mathbb{C}_{e;R} \\ \mathbb{C}^2_{e,\nu;L} \end{pmatrix} \oplus L^2(S) \otimes \begin{pmatrix} (\mathbb{C}_{d;R} \oplus \mathbb{C}_{u;R}) \otimes \mathbb{C}^{N_c} \\ \mathbb{C}^2_{d,u;L} \otimes \mathbb{C}^{N_c} \end{pmatrix} \end{aligned} \tag{2.1}$$

where S denotes the space of spinors, N_c the color degrees of freedom; and similarly for the conjugate space $\bar{\mathcal{H}} = \bar{\mathcal{H}}_\ell \oplus \bar{\mathcal{H}}_q$ of antiparticles. When right handed neutrinos are present, $\mathcal{H}_\ell$ in (2.1) is replaced by

$$L^2(S) \otimes \begin{pmatrix} \mathbb{C}_{e;R} \oplus \mathbb{C}_{\nu;R} \\ \mathbb{C}^2_{e,\nu;L} \end{pmatrix}.$$

In the old scheme there is actually a pair of algebras $(\mathcal{A}, \mathcal{B})$, with compatible actions on $\mathcal{H} \oplus \bar{\mathcal{H}}$. They are the tensor product of the commutative algebra of smooth functions over the ordinary spacetime by the finite-part algebras

$$\mathcal{A}_F := \mathbb{C} \oplus \mathbb{H}, \qquad \mathcal{B}_F := \mathbb{C} \oplus M_3(\mathbb{C}),$$

representing respectively the flavour and colour degrees of freedom; here $\mathbb{H}$ is the algebra of quaternions of Hamilton. In the new scheme they are replaced by a single algebra $\mathcal{C}$ with finite part $\mathcal{C}_F := \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$.

Denote by Ψ a generic element of the fermion space. The gauge invariant action associated to the fermion fields

$$I(\Psi) = \langle \Psi \mid (D + A_{NC})\Psi \rangle \tag{2.2}$$

gives rise both to the kinetic and the Yukawa terms in the SM Lagrangian. By construction, the old scheme theory is invariant under the direct product of the groups of unitaries of $\mathcal{A}$ and $\mathcal{B}$, which is $C^\infty(M, U(1) \times SU(2)_L \times U(1) \times U(3))$. Thus, the noncommutative philosophy faces the problem of finding a credible and useful way to reduce this group to a $C^\infty(M, SU(2)_L \times U(1)_Y \times SU(3))$ subgroup. Note that we are not allowed to change the representation of these groups, which is given by the representation of the algebras.

Actually, A_{NC} is the sum of the flavour and colour gauge potentials A_f and A_c. Let A, V, A', K be skewhermitian 1-forms with values in $\mathbb{C}$, $\mathbb{H}$, $\mathbb{C}$ and $M_3(\mathbb{C})$ respectively. One has, for each fermionic family with a right handed neutrino:

$$A_f + A_c = \begin{array}{c} \\ eR \\ \nu R \\ dR \\ uR \\ \ell L \\ qL \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} eR & \nu R & dR & uR & \ell L & qL \end{array} \\ \begin{pmatrix} A+A' & & & & & \\ & -A+A' & & & & \\ & & A+K & & & \\ & & & -A+K & & \\ & & & & V+A' & \\ & & & & & V+K \end{pmatrix} \end{array}, \tag{2.3}$$

plus a conjugate copy ($A \to -A$, $A' \to -A'$, $K \to -K$) in the antiparticle space. We are forgetting about the off-diagonal terms, which are unimportant here. If there are no right handed neutrinos, just suppress the second row and column.

Connes finds in [2] a reduction rule, called the unimodularity condition on the $\mathcal{U}(\mathcal{A}) \times \mathcal{U}(\mathcal{B})$ group, which can be rewritten [5] as

$$\mathrm{Tr}_{\mathcal{H}_R}(A_f + A_c) = 0, \qquad \mathrm{Tr}_{\mathcal{H}_L}(A_f + A_c) = 0. \tag{2.4}$$

Here $\mathcal{H}_R$ denote the space of the right-handed particles and $\mathcal{H}_L$ the space of the left-handed particles (the same conditions apply with the same result on the antiparticle side). Now $V^* = -V$ means that V is a zero-trace quaternion, so $\mathrm{Tr}_{\mathcal{H}_L}(A_f) = 0$ automatically; thus $\mathrm{Tr}_{\mathcal{H}_L}(A_c) = 0$, which yields the condition

$$A' = -\operatorname{Tr} K.$$

Let N_1 be the number of massive neutrino species. Then,

$$\mathrm{Tr}_{\mathcal{H}_R}(A_f + A_c) = (N_F + N_1)A' + (N_F - N_1)A + 2N_F \operatorname{Tr} K.$$

Combining both conditions, we get the reduction rule:

$$A = A' = -\operatorname{Tr} K. \tag{2.5}$$

as long as $N_1 < N_F$. On the other hand, if all species of neutrinos have right handed components, the abelian part A of the flavour gauge potential remains free [5]. This is to be regarded as a drawback of the old scheme.

When condition (2.5) holds, we can identify $A = A' = -\operatorname{Tr} K$ in (2.3) as the generator of the $U(1)_Y$ physical gauge field. Thus the abelian part of the noncommutative gauge potential reads:

$$\begin{array}{c} \\ eR \\ \nu R \\ dR \\ uR \\ \ell L \\ qL \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} eR & \nu R & dR & uR & \ell L & qL \end{array} \\ \begin{pmatrix} 2A & & & & & \\ & 0 & & & & \\ & & \frac{2}{3}A & & & \\ & & & -\frac{4}{3}A & & \\ & & & & A & \\ & & & & & -\frac{1}{3}A \end{pmatrix} \end{array} \oplus \begin{array}{c} \\ \bar{e}L \\ \bar{\nu}L \\ \bar{d}L \\ \bar{u}L \\ \bar{\ell}R \\ \bar{q}R \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} \bar{e}L & \bar{\nu}L & \bar{d}L & \bar{u}L & \bar{\ell}R & \bar{q}R \end{array} \\ \begin{pmatrix} -2A & & & & & \\ & 0 & & & & \\ & & -\frac{2}{3}A & & & \\ & & & \frac{4}{3}A & & \\ & & & & -A & \\ & & & & & \frac{1}{3}A \end{pmatrix} \end{array}. \tag{2.6}$$

In summary, we have killed two extra $U(1)$ fields and we get in addition the tableau of hypercharge assignments of the SM: if we conventionally adopt $Y(e_L, \nu_L) = -1$, there follows for left-handed (anti-)leptons and (anti-)quarks: $Y(\bar{e}_L) = 2$; $Y(\bar{\nu}_L) = 0$; $Y(\bar{d}_L) = \frac{2}{3}$; $Y(\bar{u}_L) = -\frac{4}{3}$ and $Y(d_L, u_L) = \frac{1}{3}$.

The matter of gauge invariance is more subtle in the new scheme. An important role is played by the antilinear isometry J that interchanges the particle and antiparticle subspaces:

$$J(\Psi, \bar{\Xi}) = (\Xi, \bar{\Psi}) \text{ for } (\Psi, \bar{\Xi}) \in \mathcal{H} \oplus \bar{\mathcal{H}}.$$

The action of the gauge group is no longer simply given by the restriction of the $\mathcal{C}$ action; rather it is of the form $(\Psi, \bar{\Xi}) \mapsto uJuJ(\Psi, \bar{\Xi})$, for u belonging to $C^\infty(M, U(1) \times SU(2)_L \times U(3))$. This translates into a noncommutative fermionic action of the form (2.2), where now $A_{NC} = \widetilde{A} + J\widetilde{A}J$ with

$$\widetilde{A} = \begin{array}{c|cccccc} & eR & \nu R & dR & uR & \ell L & qL \\ \hline eR & A &&&&& \\ \nu R && -A &&&& \\ dR &&& A &&& \\ uR &&&& -A && \\ \ell L &&&&& V & \\ qL &&&&&& V \end{array} \oplus \begin{array}{c|cccccc} & \bar{e}L & \bar{\nu}L & \bar{d}L & \bar{u}L & \bar{\ell}R & \bar{q}R \\ \hline \bar{e}L & -A &&&&& \\ \bar{\nu}L && -A &&&& \\ \bar{d}L &&& -K &&& \\ \bar{u}L &&&& -K && \\ \bar{\ell}R &&&&& -A & \\ \bar{q}R &&&&&& -K \end{array}. \tag{2.7}$$

For a fermion family without right-handed neutrino, suppress the second row and column.

Now, following Connes, instead of (2.4) we impose the single unimodularity condition

$$\operatorname{Str} A_{NC} = 0. \tag{2.8}$$

Reasoning as above, one gets again the reduction to the SM gauge group and hypercharges. Note that this happens irrespectively of whether right-handed neutrinos are present (it turns out [6] that the old $(\mathcal{A}, \mathcal{B})$ scheme with all neutrinos with right-handed components does not obey Poincaré duality —therefore is not a noncommutative manifold in the strict sense— whereas the new $\mathcal{C}$ scheme is Poincaré selfdual in any case).

3. Unimodularity = cancellation of anomalies.

In this section we shall show that either one of the two unimodularity conditions, eqs. (2.4) and (2.8) is equivalent to anomaly cancellation in the Standard Model, when obtained within the NCG framework. We will assume that each family is anomaly free by itself, and we will allow for the possibility of right-handed neutrinos towards the end of the section.

It is a fact that the unimodularity conditions [2], eqs (2.4) and (2.8) on the NCG potentials do select the representations of $SU(3) \times SU(2) \times U(1)_Y$ carried out by leptons and quarks as observed in Nature which in obvious notation is, for the $\bar{e}_L$,$l_L = (e_L, \nu_L)$, $\bar{d}_L$, $\bar{u}_L$ and $q_L = (d_L, u_L)$, respectively:

$$(1, 1, 2) \oplus (1, 2, -1) \oplus (\bar{3}, 1, 2/3) \oplus (\bar{3}, 1, -4/3) \oplus (3, 2, 1/3) \tag{3.1}$$

It is then plain that unimodularity implies absence of anomalies. Our aim in this section is to show that in a certain sense, the reverse is true as well. Let us note, first of all, that locally, $U(3) \sim SU(3) \times U(1)_K$. The index K in the abelian factor stresses the fact that it comes from the traceful generator in $U(3)$. Now, prior to imposing any unimodularity condition, the NCG formalism yields a model with a gauge symmetry which is either $G \sim SU(3) \times SU(2) \times U(1)_A \times U(1)_{A'} \times U(1)_K$ (in the old scheme), or else $G' \sim SU(3) \times SU(2) \times U(1)_A \times U(1)_K$ (in the new scheme).

Moreover, the representations are not arbitrary; rather, they are fixed by the corresponding representation of the NCG algebra over the Hilbert space of the fermions. Actually, from eq. (2.3) we learn that the fermions $\bar{e}_L$,l_L,$\bar{d}_L$,$\bar{u}_L$ and q_L transform under G as

$$(1,1,y,y',0) \oplus (1,2,0,-y',0) \oplus (\bar{3},1,y,0,k) \oplus (\bar{3},1,-y,0,k) \oplus (3,2,0,0,-k), \qquad (3.2)$$

Where y, y' and k set the $U(1)$ charge scales and do not vanish. Similarly, the G' charges in the new scheme are:

$$(1,1,2y,0) \oplus (1,2,-y,0) \oplus (\bar{3},1,y,k) \oplus (\bar{3},1,-y,k) \oplus (3,2,0,-k) \qquad (3.3)$$

All the NCG reasoning up to now has been classical. If from now on, one considers the lagrangian so obtained as a standard quantum field theory, it is obvious that one should impose cancellation of anomalies in order to get a sensible theory. It is easy to see, though [7] that this theory is always anomalous, due to the fact that, for G,

$$trY_A^3 = y^3 \qquad (3.4)$$

and the $U(1)[SU(2)]^2$ anomalies, for G', are given by:

$$trY_A\{T^a_{SU(2)}, T^b_{SU(2)}\} = -2y \qquad (3.5)$$

(Both quantities are always different from zero). This means that the group of unitaries of the algebra(s) does *not* qualify as a consistent symmetry group at the quantum level (and this is true in both the old and the new formulations). A natural question to ask at this stage is whether there are subgroups of G or G' that are anomaly free (with the representation content induced from the embedding). We shall actually deal with a more modest version of the preceding, namely we shall assume that the subgroups are of the form $SU(3)\times SU(2)\times H$, with the same quantum numbers for $SU(3)\times SU(2)$ as indicated in eqs. (3.1), (3.2) and (3.3). Although those are certainly the simplest possibilities, they do not exhaust them all. It is a quite natural restriction from the NCG point of view, though, because at any rate both the color and weak isospin structure are imposed by hand precisely for them to coincide with the standard model and it does not seem wise to tamper with them more than necessary. This leaves room for only two possibilities for H: either $U(1)$ or else $U(1) \times U(1)$ (in the new scheme there is only the former possibility).

To be specific, what we are going to prove is that the *unique* subgroup of G (G'), under the restrictions just specified, which is anomaly free, is precisely, the standard model group

, with the physical representation content. The actual embedding is defined, in the old scheme, through an identification of A, A' and K, according to (2.5); and in the new scheme, owing to the identification of A and K conveyed by (2.8).

In the old scheme there are two possibilities for H, namely $U(1) \times U(1)$ and $U(1)$. There are three different ways of getting the first possibility, to wit (representing the two abelian gauge fields of H by B and B' , and denoting by $\hat{K} =: \frac{trK}{3}$):

$$
\begin{aligned}
i) A' &= \alpha A + \beta \hat{K} \\
B &= A \\
B' &= K \\
ii) \hat{K} &= \gamma A \\
B &= A \\
B' &= A' \\
iii) A &= 0 \\
B &= \hat{K} \\
B' &= A'
\end{aligned}
\tag{3.6}
$$

The linear constraints (3.6) now determine the H quantum numbers in terms of two arbitrary real parameters, x and y, namely

$$i)((1+\alpha)x, \beta y) \oplus (-\alpha x, -\beta y) \oplus (x, y) \oplus (-x, y) \oplus (0, -y), \tag{3.7}$$

$$ii)(x, y) \oplus (0, -y) \oplus ((1+\gamma)x, 0) \oplus ((\gamma - 1)x, 0) \oplus (-\gamma x, 0), \tag{3.8}$$

$$iii)(0, y) \oplus (0, -y) \oplus (x, 0) \oplus (x, 0) \oplus (-x, 0), \tag{3.9}$$

It is now a simple exercise to show that (3.7) is anomalous, since

$$
\begin{aligned}
TrY_B^3 &= ((1+\alpha)^3 - 2\alpha^3)x^3 \\
TrY_B\{T^a_{SU(2)}, T^b_{SU(2)}\} &= -2\alpha x,
\end{aligned}
\tag{3.10}
$$

and both expressions cannot vanish simultaneously, since $x \neq 0$. The same thing happens with the representation conveyed by (3.8):

$$TrY_{B'}^3 = -y^3$$

(always non-zero) as well as with the one coming from (3.9):

$$TrY_{B'}^3 = -y^3$$

(again, never zero).This means that (3.6) never leads to an anomaly free representation.

Let us now examine the other option, $H = U(1)$. The three different possibilities for getting $H = U(1)$ are now (representing by B the only remaining abelian gauge field):

$$\begin{aligned} i) A &= \alpha A' \\ \hat{K} &= \beta A' \\ B &= A' \\ ii) A &= \alpha \hat{K} \\ A' &= 0 \quad , \\ B &= \hat{K} \\ iii) A' &= 0 \\ \hat{K} &= 0 \\ B &= A \end{aligned} \tag{3.11}$$

The charges are then given in terms of a single parameter, $x \neq 0$:

$$i)(1+\alpha)x \oplus -x \oplus (\alpha+\beta)x \oplus (\beta-\alpha)x \oplus -\beta x, \tag{3.12}$$

$$ii)\alpha x \oplus 0 \oplus (1+\alpha)x \oplus (\alpha-1)x \oplus -x, \tag{3.13}$$

$$iii)x \oplus 0 \oplus x \oplus -x \oplus 0, \tag{3.14}$$

Now it is easily checked that (3.13) and (3.14) are both anomalous, since, for example, (3.13) implies: $TrY_B\{T^a_{SU(2)}, T^b_{SU(2)}\} = -6x \neq 0$, and (3.14) leads in turn to: $TrY_B^3 = x^3 \neq 0$. On the other hand, for (3.12) one gets

$$\begin{aligned} TrY_B^3 &= ((1+\alpha)^3 - 2 + 18\alpha^2\beta)x^3 \\ TrY_B\{T^a_{SU(2)}, T^b_{SU(2)}\} &= 2(1+3\beta)x, \end{aligned} \tag{3.15}$$

This means that (3.12) is anomaly-free if and only if

$$\alpha = 1; \quad \beta = -1/3, \tag{3.16}$$

This correspond to the fermionic representation of $SU(3) \times SU(2) \times U(1)$:

$$(1,1,2x) \oplus (1,2,-x) \oplus (\bar{3},1,2x/3) \oplus (\bar{3},1,-4x/3) \oplus (3,2,x/3)$$

which coincides with the standard representation, up to normalization. Note that if we substitute $\alpha = 1$ and $\beta = -1/3$ back in eq. (3.11) one obtains $A = A' = -trK$, so that one recovers the unimodularity constraints given in eq (2.5).

In the new scheme, the group of unitaries is G', with representation content given by (3.3).With a by now familiar reasoning, there are two ways of getting $H = U(1)$ as a subgroup, namely:

$$\begin{aligned} i)A &= 0 \\ B &= \hat{K} \\ ii)\hat{K} &= \gamma A \\ B &= A \end{aligned} \tag{3.17}$$

(where B is the remaining abelian gauge field). The case (3.17i) carries the following representation of $SU(3) \times SU(2) \times U(1)_B$ (in terms of a real parameter $x \neq 0$):

$$(1,1,0) \oplus (1,2,0) \oplus (\bar{3},1,x) \oplus (\bar{3},1,x) \oplus (3,2,-x) \tag{3.18}$$

Now it is plain that this is anomalous, because, for example, $TrY_A\{T^a_{SU(2)}, T^b_{SU(2)}\} = -6x \neq 0$

On the other hand, among the representations given by (3.17ii), and parametrized by γ, there is a unique one which enjoys the property of being anomaly free, because then the $U(1)$ fermion quantum numbers are

$$2x \oplus -x \oplus (1+\gamma)x \oplus (\gamma-1)x \oplus -\gamma x \tag{3.19}$$

so that the condition of vanishing of $TrY_B^3 = 6 + 18\gamma$ uniquely fixes $\gamma = -1/3$. This yields again the anomaly free representation given before in eq. (3.1). If we substitute $\gamma = -1/3$ in eq. (3.19ii), one obtains $A = -trK$, namely the unimodularity condition.

We shall now, for the sake of completeness, generalize the preceding discussion to the case in which there exists a right handed neutrino. In the old NCG scheme the fermions (namely, now $\bar{e}_L, \bar{\nu}_L, l_L, \bar{d}_L, \bar{u}_L, q_L$) carry the following representation of G:

$$(1,1,y,y',0)\oplus(1,1,-y,y',0)\oplus(1,2,0,-y',0)\oplus(\bar{3},1,y,0,k)\oplus(\bar{3},1,-y,0,-k)\oplus(3,2,0,0,-k) \tag{3.20}$$

It is readily seen that this representation is anomalous, because $TrY_{A'}\{T^a_{SU(2)}, T^b_{SU(2)}\} = -2y' \neq 0$. We next look for subgroups of G having the $SU(3) \times SU(2)$ structure given by eq. (3.2). There are again only two possibilities, either $H = U(1) \times U(1)$ or else $H = U(1)$. The representations of $U(1) \times U(1)$ induced by (3.20) are obtained by imposing on the abelian gauge fields A, A' and $\hat{K}$, the linear restrictions spelled out in (3.6). It is not difficult to see that the subcases (3.6ii) and always lead to anomalous representations. This result is indeed the same as in the massless neutrino situation. However, at variance with this case, there is now an anomaly free representation in the subcase (3.6i). Actually, (3.6i) leads to the following representations of $H = U(1)_B \times U(1)_{B'}$:

$$((1+\alpha)x, \beta y) \oplus ((\alpha-1)x, \beta y) \oplus (-\alpha x, -\beta y) \oplus (x,y) \oplus (-x,y) \oplus (0,-y) \tag{3.21}$$

Absence of anomalies implies $TrY_B^3 = 6\alpha x^3 = 0$ and $TrY_{B'}Y_B^2 = (2\beta+6)yx^2 = 0$, which is only possible if $\alpha = 0$ and $\beta = -3$. (Recall that both $x, y \neq 0$, because they set the charge scale). Now, if one substitutes this in (2.23), and works out the remaining anomaly

constraints (including mixed gauge-gravitational anomalies), one easily checks that they all hold. Besides, the hypercharge assignments are again exactly the same as the ones obtained using unimodularity. Plugging the values for α and β back in (3.6) we obtain the constraint $A' = -trK$, while A remains free. (All this is the same as unimodularity [5]). Arguing from the standpoint of absence of anomalies, we have shown that, within the old scheme, we need at least a family with no right-handed neutrino, if reduction to the standard model representation for $SU(3) \times SU(2) \times U(1)$ is to be achieved. Exactly the same conclusion can be reached from the standpoint of unimodularity [5].

To close this section, we shall comment on the equivalence between unimodularity and absence of anomalies with right handed neutrinos in the framework of the new scheme. Performing exactly the same type of analysis as we did repeatedly, one again reaches the conclusion that both viewpoints are equivalent, and both reduce the gauge group from $SU(3) \times SU(2) \times U(1)_A \times U(1)_K$ down to the standard model group $SU(3) \times SU(2) \times U(1)$ with the correct representation content.

4. Conclusions and comments

Non-Commutative Geometry has provided us, up to now, with a way of getting *some* lagrangians in field theory, corresponding to the standard model, with certain relationships among the parameters; not every set of coupling constants can be obtained from this viewpoint [1],[2],[3]. If one, however, now takes this *classical* lagrangian as the starting point for a *quantum* theory, these relationships are not maintained (because they are not first integrals of the renormalization group)[9],[10]. In the present work, we have pointed out at a curious fact, *unimodularity* , which, being as it is a mathematical restriction on the group of unitaries of a certain algebra, has been shown to be equivalent to the *physical* condition of absence of anomalies in the model.

We would like now to make some comments concerning the matter of anomaly cancellation in the NCG framework. We should stress that the constraints implying cancelation of anomalies involving gravitons have never been used. In the NCG formulation of the Standard Model, once anomaly freedom for the representation of the gauge group is achieved, anomaly freedom of the theory coupled to gravity follows. This is at variance with the ordinary derivation of the Standard Model using standard differential geometry, where, actually, the cancelation of the triangle anomaly involving a $U(1)$ field and two gravitons is needed to restrict the allowed hypercharges as much as possible [8]. Finally, it is remarkable that the bizarre solution to the anomaly freedon equations found by Minaham et al. [8] does not occur in the NCG reconstruction of the Standard Model. The bizarre solution is given by the following hypercharge, Y, assignments

$$Y(\bar{e}_L) = Y(l_L) = Y(q_L) = 0, \;\; Y(\bar{d}_L) = -Y(\bar{u}_L) \neq 0$$

For these assignments to occur in NCG the following linear relations amomg the $U(1)$ fields in eq. (2.3) should hold

$$A + A' = 0, \;\; A' = 0, \;\; TrK = 0,$$

The fermions $\bar{e}_L$, l_L and q_L have thus vanishing hypercharges. But then the preceding linear relations imply

$$A + TrK = 0, \;\; -A + TrK = 0,$$

so that $\bar{d}_L$ and $\bar{u}_L$ have unavoidably zero hypercharge. The bizarre solution thus evaporates. A similar argument can be devised to explain the lack of bizarre solution in the new scheme. We would like to finish this article by saying that the results presented in it hint at a deeper relationship between quantum physics and NCG than was thought before. It certainly remains a fascinating avenue to explore further.

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